高一數學

一、單選題：(每題 5 分)

1.	2.	3.	4.
(1)	(5)	(4)	(4)

二、多重選擇題：(每題 6 分，錯 1 個選項得 4 分，錯 2 個選項得 2 分，錯 3 個以上選項或未作答皆不給分。)

1.	2.	3.
(1)(3)(5)	(1)(5)	(2)(4)

三、填充題：(每格 5 分，共 50 分)

1.	2.	3.	4.
$21-5\sqrt{3}$	$-\frac{2}{3}$	$\frac{\sqrt{1-k^2}}{-k}$	$d>b>a>c$
5.	**6.**	**7.**	**8.**
45	$\frac{240\sqrt{7}}{7}$	$\sqrt{65}$	18
9.	**10.**		
49	$\frac{117}{14}$		

四、計算題：(共 12 分，請詳列算式，否則不予給分)

1.

(1) $\cos\angle BAC=\frac{3^2+4^2-(\sqrt{7})^2}{2(3)(4)}=\frac{3}{4}$ (2 分)

$\cos\angle EAG=\cos(180°-\angle BAC)=-\frac{3}{4}$ (2 分)

$\overline{EG}^2=3^2+4^2 2(3)(4)\cos\angle EAG=43 \Rightarrow \overline{EG}=\sqrt{43}$ (2 分)

(2) $\because \sin\angle EAG=\sqrt{1-\cos^2\angle EAG}=\sqrt{1-(\frac{-3}{4})^2}=\frac{\sqrt{7}}{4}$ (3 分)

又 $\frac{\overline{EG}}{\sin\angle EAG}=2R$，故 $R=\frac{\sqrt{43}}{\sqrt{7}/4}\cdot\frac{1}{2}=\frac{2\sqrt{301}}{7}$ (3 分)

一、單選題：

1. 原式 $=-1+(\frac{1}{\sqrt{3}})^2=-\frac{2}{3}$

2. 令 θ_1、θ_2 分別為 L_1、L_2 之斜角，$\tan\theta_1=-\frac{1}{\sqrt{3}} \Rightarrow \theta_1=-30°$，$\tan\theta_2=1 \Rightarrow \theta_2=45°$，
故 $\theta_2-\theta_1=75°$

3. 設 $\overline{CH}=k$ $\because \tan C=2$ $\therefore \overline{AH}=2k$ $\because \cos B=\frac{3}{5}$ $\therefore \overline{AB}=\frac{5}{2}k$，$\overline{BH}=\frac{3}{2}k$
由 $\frac{3}{2}k-3=k+3 \Rightarrow k=12$，故 $\overline{AB}=30$

4. $\frac{\overline{AP}}{\sin B}=2R_1$，$\frac{\overline{AC}}{\sin B}=2R_3 \Rightarrow \frac{\overline{AP}}{\overline{AC}}=\frac{3}{5}$; $\frac{\overline{AP}}{\sin C}=2R_2$，$\frac{\overline{AB}}{\sin C}=2R_3 \Rightarrow \frac{\overline{AP}}{\overline{AB}}=\frac{1}{5}$
$\therefore \overline{AB}:\overline{AP}:\overline{AC}=15:3:5$

二、多重選擇題：

1. (2) $\cos(180°-\theta)=-\cos\theta\neq\overline{OE}$ (4) $\overline{CD}=\frac{1}{\tan\theta}$

2. (1) $\overline{BD}^2=3^2+8^2-2(3)(8)\cos 60°=49 \Rightarrow \overline{BD}=7$

(2) $\cos 120°=\frac{\overline{CD}^2+3^2-7^2}{2\cdot 3\cdot\overline{CD}}=-\frac{1}{2} \Rightarrow \overline{CD}=5$

(3) $\frac{\overline{BD}}{\sin 60°}=2R \Rightarrow R=\frac{7}{\sqrt{3}/2}\cdot\frac{1}{2}=\frac{7}{\sqrt{3}}$，故圓面積為 $\frac{49\pi}{3}$

(4) $\frac{3}{\sin\angle ADB}=\frac{7}{\sin 60°} \Rightarrow \sin\angle ADB=\frac{3\sqrt{3}}{14}<\frac{1}{2} \Rightarrow \angle ADB<30°$
故 $\angle ADC=2\angle ADB<60°$

(5) $ABCD$ 面積 $=\Delta ABD+\Delta BCD=\frac{1}{2}\cdot 3\cdot 8\cdot\sin 60°+\frac{1}{2}\cdot 3\cdot 5\cdot\sin 120°=\frac{39\sqrt{3}}{4}$

3. (1) $2A=2B$ 或 $2A+2B=180° \Rightarrow$ 等腰或直角三角形

(3) $a\cdot\frac{b^2+c^2-a^2}{2bc}=b\cdot\frac{a^2+c^2-b^2}{2ac} \Rightarrow (a^2-b^2)(c^2-a^2-b^2)=0 \Rightarrow$ 等腰或直角三角形

(4) $\frac{a}{2R}=2\cdot\frac{a^2+c^2-b^2}{2ac}\cdot\frac{c}{2R} \Rightarrow b=c \Rightarrow$ 等腰三角形

(5) $(b-c)(1-\sin^2 A)=b(1-\sin^2 B)-c(1-\sin^2 C)$
$(b-c)(1-(\frac{a}{2R})^2)=b(1-(\frac{b}{2R})^2)-c(1-(\frac{c}{2R})^2)$
$(b-c)(b^2+bc+c^2-a^2)=0 \Rightarrow$ 等腰或 $\angle A=120°$ 之鈍角三角形

三、填充題：

1. $\Delta ABC=\Delta OAB+\Delta OBC-\Delta OAC=\frac{1}{2}\cdot 4\cdot 6\cdot\sin 30°+\frac{1}{2}\cdot 6\cdot 5\cdot\sin 90°-\frac{1}{2}\cdot 4\cdot 5\cdot\sin 120°=21-5\sqrt{3}$

2. $6\cos\theta-\frac{\sin\theta}{\cos\theta}=\frac{4}{\cos\theta} \Rightarrow 6(1-\sin^2\theta)-\sin\theta=4 \Rightarrow \sin\theta=\frac{1}{2}$ 或 $-\frac{2}{3}$

$\because \theta$ 是第四象限角 $\therefore \sin\theta = -\frac{2}{3}$

3. $\cos(-100°) = -\cos 80°$ $\therefore \cos 80° = -k$ 故 $\tan 970° = \tan 70° = \frac{\sqrt{1-k^2}}{-k}$

4. $a = \sin 280° = -\sin 80°$ $\Rightarrow$ $-1 < a < 0$

$b = \cos 661° = \cos 59° = \sin 31°$ $\Rightarrow$ $0 < b < 1$

$c = \tan 1034° = -\tan 46°$ $\Rightarrow$ $c < -1$

$d = \tan(-329°) = \tan 31°$ $\Rightarrow$ $d > 0$ ，故 $d > b > a > c$

5. $\overline{BC}^2 = 2^2 + (1+\sqrt{3})^2 - 2(2)(1+\sqrt{3})\cos 30° = 2$ $\Rightarrow$ $\overline{BC} = \sqrt{2}$

$\cos C = \frac{\sqrt{2}^2 + (1+\sqrt{3})^2 - 2^2}{2\cdot\sqrt{2}(1+\sqrt{3})} = \frac{1}{\sqrt{2}}$ $\Rightarrow$ $\angle C = 45°$

6. 三邊長比為 $\frac{1}{15}:\frac{1}{12}:\frac{1}{10} = 4:5:6$，設三邊長為 $4k$、$5k$、$6k$

ΔABC 面積 $= \frac{4k\cdot 15}{2} = \sqrt{\frac{15k}{2}\frac{7k}{2}\frac{5k}{2}\frac{3k}{2}}$ $\Rightarrow$ $k = \frac{8}{\sqrt{7}}$ ，故面積為 $30k = \frac{240\sqrt{7}}{7}$

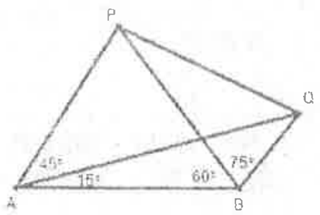
7. 作 $\overline{ID} \perp \overline{BC}$ ，此時 $\overline{BD} = \frac{15+13+14}{2} - 14 = 7$ ，$\overline{ID} = \Delta ABC$ 的內切圓半徑 r

$\because \Delta ABC$ 面積 $= \sqrt{21\cdot 6\cdot 8\cdot 7} = 84$ ，$\therefore r = \frac{\Delta}{s} = \frac{84}{21} = 4$ ，故 $\overline{BI} = \sqrt{7^2+4^2} = \sqrt{65}$

8. $\frac{\overline{AQ}}{\sin 135°} = \frac{18}{\sin 30°}$ $\Rightarrow$ $\overline{AQ} = 18\sqrt{2}$

$\because \Delta ABP$ 為正三角形 $\therefore \overline{AP} = 18$

$\therefore \overline{PQ}^2 = 18^2 + (18\sqrt{2})^2 - 2(18)(18\sqrt{2})\cos 45° = 324$ $\Rightarrow$ $\overline{PQ} = 18$



9. 設正方形邊長為 x ，令 $\angle ABP = \theta$ ，$\angle PBC = 90° - \theta$

$\cos\theta = \frac{x^2 + 5^2 - (4\sqrt{2})^2}{2\cdot x\cdot 5} \cdots ①$ $\cos(90° - \theta) = \sin\theta = \frac{5^2 + x^2 - (3\sqrt{2})^2}{2\cdot x\cdot 5} \cdots ②$

由上兩式平方和為 1， 可得 $(\frac{x^2-7}{10x})^2 + (\frac{x^2+7}{10x}) = 1$ $\Rightarrow$ $x^2 = 1$（不合）或 49

10. $\sin\angle DAC = \sin(90° - \angle BAD) = \cos\angle BAD = \frac{3}{5}$

$\frac{1}{2}\cdot 5\cdot\overline{AD}\cdot\sin\angle BAD + \frac{1}{2}\cdot\overline{AD}\cdot 12\sin\angle DAC = \frac{1}{2}\cdot 5\cdot 12\cdot\sin\angle BAC$ $\Rightarrow$ $\overline{AD} = \frac{75}{14}$

又 $\frac{\overline{CD}}{\sin\angle DAC} = \frac{\overline{AD}}{\sin\angle ACD}$ $\Rightarrow$ $\frac{\overline{CD}}{3/5} = \frac{75/14}{5/13}$ $\Rightarrow$ $\overline{CD} = \frac{117}{14}$